Letters

Comments on "Power-Handling Capability of the Rhombic Waveguide"

Patricio A. A. Laura and Liberto Ercoli

The purpose of the present correspondence is two fold: first to congratulate the author for his important contribution [1] and the second to present some considerations regarding the solution of Helmholtz equation as well as other wave-type equations in rhombic domains.

It seems possible to use Overfelt's approach in unsteady diffusion problems in rhombic domains. This is certainly an interesting possibility since no analytical solutions are available in that area. On the other hand several publications which deal with vibrating rhombic plates are available in the open literature [2]–[4]. The conformal mapping approach has been applied in [2]–[5] and also in [6] in order to obtain the fundamental cut-off frequency of a waveguide of rhombic boundary with a concentric circular hole (TM modes).

References

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Reply to Comments on "Power-Handling Capability of the Rhombic Waveguide"

P. L. Overfelt and C. S. Kenney

The authors would like to thank Patricio A. A. Laura and Liberto Ercoli for their comments and suggestions on the rhombic waveguide problem [1]. Since the time period in which that work was completed, we have found that the infinite series of nonseparable solutions technique can be applied to most linear partial differential equations with constant coefficients. Thus boundary value problems for complex geometries governed by the Laplace, diffusion, free-particle Schrodinger, etc. equations can be solved using this method. However we have always assumed the usual Dirichlet and Neumann boundary conditions and do not know how the method would perform for more complicated boundary conditions. The idea of applying our approach to unsteady diffusion problems in rhombic domains is a fascinating one.

For many years we have been aware of Professor Laura's work on eigenvalues of waveguides with complicated geometries [2], [3] but were unaware of his work on rhombic plates and membranes (see above [2], [5], and [6]). We are grateful for having these references pointed out.

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Comments on "Spectral-Domain Computation of Characteristic Impedances and Multiport Parameters of Multiple Coupled Microstrip Lines"

Smain Amari

In the above mentioned paper [1] the authors present an extensive discussion of the problem of systems of coupled lines. The spectral domain technique is used to extract the dispersion relations of the structure along with the corresponding current densities on the different lines. However, in equation (7) page 216, they state that the eigencurrent matrix $[M_I]$ and the corresponding eigenvoltage matrix $[M_v]$ are related by

$$[M_v] = [[M_I]^T]^{-1}. \qquad (1)$$

(Equations in this article are referred to as eqn.n whereas those in the authors's paper as equation (n)). I should hasten to say that this is not the first time this equation has been mentioned. Indeed Marx [2] states, as do the authors, that this relation holds if the both eigencurrents and eigenvoltages are normalized (for each mode). This mathematical statement cannot be disputed. The point that the authors failed to see is to examine the meaning of the normalization itself for the problem under consideration. Is it POSSIBLE to normailize both currents and voltages SIMULTANEOUSLY?. For the

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sake of argument let us assume that we indeed could achieve such a simultaneous normalization. If we then use the normalized matrices $[M_I]$ and $[M_v]$ to expand arbitrary current and voltage distributions, we are led to an expression of the form [2], [3]:

$$[Z_c] = [M_v][M_I]^{-1}. \tag{2}$$

Here $[Z_c]$ is what is usually referred to as 'the total characteristic impedance matrix'. In [2], it is rather the inverse of $[Z_c]$ which is defined explicitly. If equation (7) is used in equ.2 we get

$$[Z_c] = [[M_I]^T]^{-1}[M_I]^{-1}. \tag{3}$$

Equ.3 is a very powerful statement. To see what this means let us consider two symmetric lines, ANY two symmetric lines. It is well known that such a structure allows an odd and an even mode. The even mode corresponds to equal currents on the lines while the odd to opposite currents. The corresponding normalized eigencurrent and eigenvoltage matrices can therefore be written as follows (assuming that equation (7) in the authors' paper holds):

$$[M_I] = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix} \tag{4}$$

$$[M_v] = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{pmatrix}. \tag{5}$$

If we now use these two matrices to calculate the corresponding $[Z_c]$ matrix we get

$$[Z_c] = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{6}$$

The 2 × 2 unit matrix! This is saying that ANY two symmetric coupled lines, regardless of their physical and geometrical properties have the same total characteristic impedance matrix. It is also diagonal, which means that no couplers can be built using two symmetric coupled lines. In fact, all structures belonging to the same symmetry group (in the language of group theory) will have the same total characteristic impedance matrix if equation (7) were true. This obviously is in total disagreement with experimental facts. It is possible to go on citing inconsistent results due to the simultaneous normalization, but I only mention that equation (7) is dimensionally incorrect. I would certainly appreciate if the authors could clarify the above point. On a more fundamental level, this raises interesting questions. What is the physical meaning of normalizing an eigencurrent or an eigenvoltage? What is the corresponding inner product and what is its interpretation? Do a normalized eigencurrent vector and its corresponding normalized eigenvoltage vector satisfy the transmission line equations?. Direct substitution shows that the answer is no. In another section of the paper the authors calculate the characteristic impedances of the lines using 'partial powers' associated with the different lines.

$$z_{lm} = \frac{P_{lm}}{I_{lm}^2} \tag{7}$$

'P_{lm} is calculated by integrating the Poynting vector over the total cross section when the current distribution on line l corresponds to the solution for currents when $\beta = \beta_m$ and all the other line currents are zero.' Again this way of splitting the power among the different lines has been used repeatedly (even though usually the electric field is calculated with all currents present and the magnetic field with only one current non zero [3]). It is important to keep in mind that the integral of a Poynting vector represents a power flow ONLY if the electric and magnetic fields used are solutions to Maxwell's equations. This means that they DO satisfy the boundary conditions of the problem. Do the 'partial fields' obtained by setting all the currents but one to zero satisfy the boundary conditions of the problem? Certainly not. A normal mode IMPOSES a DEFINITE relationship between the currents (current densities) on the different lines in order to propagate. Any alteration of this relationship between the currents VIOLATES the boundary conditions. Setting all the currents but one to zero is clearly an alteration of the relationship imposed by the normal mode. For example nothing, *a priori*, prevents the magnetic field due to ONLY the current on one line from having a nonzero normal component on the other lines. It is now known that the characteristic impedances defined using this approach do not satisfy reciprocity [3]. This is expected since reciprocity is a property of Maxwell's equations and holds only for fields satisfying them. The authors also state that the sum of these powers is equal to the total power of the mode. This would be true if there were no coupling between the fields (currents), but this is not the case. If one considers the field lines (assuming a quasi TEM situation) of two coupled symmetric lines, it seems that for the even mode the sum of the partial powers should be larger than the total power and is smaller for the odd mode. Could the authors give details about this point (numerically if possible).

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Reply to Comments on "Spectral-Domain Computation of Characteristic Impedances and Multiport Parameters of Multiple Coupled Microstrip Lines"

V. K. Tripathi and H. Lee

There seem to be two main issues raised in the above comments: One dealing with the definition and meaning of characteristic impedance for general multiconductor systems and the other deal-

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ing with the computation of the elements of the line mode impedance matrices as implemented in [1]. We will attempt to clarify both issues.

Unlike the single line case where the characteristic impedance is defined in an unique unambiguous manner, three useful basic definitions for characteristic impedances have been used in the past for multiple coupled line structures [2]. These are:

Line Mode Impedance Matrix $\mathbf{Z}_{\text{lm}}$: The elements of this matrix represent the ratio of the voltage of line l to the current on the same line for a given mode m traveling in $+z$ direction. The lines must be terminated in these impedances in order to match the lines when this mode is excited. These impedance matrices can be readily used to evaluate the multiport immittance and other network functions as demonstrated in [3] and [4] for lower order systems and [5], [6] for the general case. These impedance matrix elements are also compatible with full wave dynamic analysis of multiconductor system and are calculated from the eigenvalues, current eigenvectors and the total power associated with the normal modes of the system as shown in [1]. It should be noted that this matrix is unique for a given system, is in general not symmetrical and can have positive as well as negative elements.

Decoupled Modal Impedances $\mathbf{Z}_0$: This is a diagonal matrix and represents the characteristic impedances of the normal modes for the decoupled transmission line system. These are used in the equivalent circuit representation of the multiconductor system such as the SPICE model used in [7]. These impedances of the decoupled lines are not unique and depend on how the voltage and current eigenvectors corresponding to the voltage and current eigenvector matrices $\boldsymbol{M}_V$ and $\boldsymbol{M}_I$ are normalized.

Characteristic Termination Impedance Matrix $\mathbf{Z}_{\text{OT}}$: It is the open circuit impedance of an N port network that perfectly terminates all the lines for all the modes, i.e., any arbitrary excitation on the multiconductor system is matched by this impedance matrix. This is perhaps the definition which is most similar to the one used for a single transmission line. This matrix is obviously symmetrical, dominant and represents a unique realizable passive network.

It should be noted that the term "the total characteristic impedance matrix" used in the comments corresponds to the third definition and that the characteristic impedance matrices defined above are interrelated. As an example, the expressions for the above characteristic impedance matrices for the simple case of two coupled lines is given by

$$\mathbf{Z}_{\text{lm}} = \begin{bmatrix} Z_{c1} & Z_{\pi 1} \\ Z_{c2} & Z_{\pi 2} \end{bmatrix}, \tag{1}$$

$$\mathbf{Z}_0 = \begin{bmatrix} Z_{c1} \dfrac{-R_\pi}{1+R_\pi^2}(R_c - R_\pi) & 0 \\ 0 & Z_{\pi 1} \dfrac{R_c}{1+R_c^2}(R_c - R_\pi) \end{bmatrix}, \tag{2}$$

$$\mathbf{Z}_{\text{OT}} = \frac{1}{R_c - R_\pi} \begin{bmatrix} R_c Z_{\pi 1} - R_\pi Z_{c1} & Z_{c2} - Z_{\pi 2} \\ Z_{c2} - Z_{\pi 2} & R_c Z_{c2} - R_\pi Z_{\pi 2} \end{bmatrix}, \tag{3}$$

where the normalized voltage eigenvector matrix is

$$\boldsymbol{M}_V = \begin{bmatrix} \dfrac{1}{\sqrt{1+R_c^2}} & \dfrac{1}{\sqrt{1+R_\pi^2}} \\ \dfrac{R_c}{\sqrt{1+R_c^2}} & \dfrac{R_\pi}{\sqrt{1+R_\pi^2}} \end{bmatrix}, \tag{4}$$

$Z_{c1,2}$ and $Z_{\pi 1,2}$ are the impedances of lines 1 and 2 for the two modes c and π, respectively [3]. For the symmetrical case of identical lines c is even and π is odd mode or $R_c = -R_\pi = 1$ and then $Z_{c1} = Z_{c2} = Z_e$ and $Z_{\pi 1} = Z_{\pi 2} = Z_o$, or

$$\mathbf{Z}_{\text{lm}} = \begin{bmatrix} Z_e & Z_o \\ Z_e & Z_o \end{bmatrix}; \quad \mathbf{Z}_0 = \begin{bmatrix} Z_e & 0 \\ 0 & Z_o \end{bmatrix} \text{ and}$$

$$\mathbf{Z}_{\text{OT}} = \tfrac{1}{2} \begin{bmatrix} Z_e + Z_o & Z_e - Z_o \\ Z_e - Z_o & Z_e + Z_o \end{bmatrix}. \tag{5}$$

The voltage and current eigenvector matrices $\boldsymbol{M}_V$ and $\boldsymbol{M}_I$ in general consist of the eigenvectors associated with product matrices **LC** and it's transpose respectively and can be calculated independently for lossless quasi TEM case in terms of the $\boldsymbol{C}$ and $\boldsymbol{L}$ matrices. However, with the full wave computations, the characterizing $2N$ port immittance matrix of the coupled line multiport or its equivalent circuit (e.g., SPICE) model [7] parameters can be evaluated in terms of the line currents and equivalent voltages. Normalized line currents and total power are readily computed for all the normal modes. The voltage and current vectors are biorthogonal or

$$\boldsymbol{V}_m^T \boldsymbol{I}_j = \sum_{k=1}^{N} V_{\text{km}} I_{kj} = \begin{cases} 0, & k \neq m, \\ P_m, & j = m, \end{cases} \tag{6}$$

where subscript k refers to the line and m and j are the normal modes. P_m is the total power associated with the mth mode. The above eigenvectors, $\boldsymbol{V}_m$ and $\boldsymbol{I}_m$, representing the voltage and current distribution for a given mode m are not independent (6).

Again, the total power associated with a given mode m is expressed in terms of the line currents and equivalent voltages corresponding to that mode as

$$P_{m,\text{total}} = \sum_{k=1}^{N} V_{\text{km}} I_{\text{km}} = V_{\text{lm}} I_{\text{lm}} \left(1 + \sum_{\substack{k=1 \\ k \neq l}}^{N} \frac{V_{\text{km}}}{V_{\text{lm}}} \frac{I_{\text{km}}}{I_{\text{lm}}} \right)$$
$$= V_{\text{lm}} I_{\text{lm}} \left(1 + \sum_{\substack{k=1 \\ k \neq l}}^{N} \frac{m_{\text{km},v}}{m_{\text{lm},v}} \frac{m_{\text{km},i}}{m_{\text{lm},i}} \right) \tag{7}$$

where, $m_{\text{km},i}$ and $m_{\text{km},v}$ are the elements of the current and voltage eigenvector matrices $\boldsymbol{M}_I$ are $\boldsymbol{M}_V$, respectively. In ref. [1] $m_{\text{km},i}$ is the kth component of the normalized current eigenvector and $m_{\text{km},v}$ are calculated from (7) in [1]. The partial line mode power used to calculate the line mode impedance matrix elements is then

$$P_{\text{lm}} = \frac{P_{m,\text{total}}}{1 + \sum\limits_{\substack{k=1 \\ k \neq l}}^{N} \dfrac{m_{\text{km},v}}{m_{\text{lm},v}} \dfrac{m_{\text{km},i}}{m_{\text{lm},i}}}. \tag{8}$$

Only the ratios of the components of current and voltage eigenvectors are used in the computations making the calculations independent of the magnitude of the eigenvectors. Then the elements of $\mathbf{Z}_{\text{lm}}$ are calculated by using $P_{\text{lm}}/I_{\text{lm}}^2$ definition. For the case of symmetrical coupled lines power is equally divided between the two lines for both the modes (8).

We hope that we have answered all the questions raised by Amari that pertain to [1].

References

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Corrections to "Modeling Three-Dimensional Discontinuities in Waveguides using Non-Orthogonal FDTD Algorithm"

Jin-Fa Lee, Robert Palandech, and Raj Mittra

In the above paper[1] there were several typographical errors:

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[1] J.-F. Lee, R. Palendech, and R. Mittra, *IEEE Trans. Microwave Theory Tech.*, vol. 40, no. 2, pp. 346- 352, Feb. 1992.

- In Eqs. (24), (25), g_{lm} should be replaced by g^{lm}.
- Equation (30) should be modified as

(a) Two-Dimensional Case:

$$\|\nabla\| = \sup_E \frac{\|\nabla E\|}{\|E\|}$$

$$= 2\sqrt{\frac{1}{(\Delta_x)^2} + \frac{1}{(\Delta_y)^2}}$$

$$\Rightarrow \Delta_t \leq \frac{1}{\sqrt{\frac{1}{(\Delta_x)^2} + \frac{1}{(\Delta_y)^2}}}$$

(b) Three-Dimensional Case:

$$\|\nabla\| = \sup_E \frac{\|\nabla E\|}{\|E\|}$$

$$= 2\sqrt{\frac{1}{(\Delta_x)^2} + \frac{1}{(\Delta_y)^2} + \frac{1}{(\Delta_x)^2}}$$

$$\Rightarrow \Delta_t \leq \frac{1}{\sqrt{\frac{1}{(\Delta_x)^2} + \frac{1}{(\Delta_y)^2} + \frac{1}{(\Delta_x)^2}}}$$

Furthermore, in Fig. 13 of the paper, the oscillations observed in the measurements were due to the mismatches at the input/output ports; likewise, the oscillations in the non-orthogonal FDTD results are attributable to the imperfect absorbing boundary conditions (ABCs). We thank Prof. C. H. Chan at University of Washington in Seattle for bringing this matter to our attention.